An analytical approach to estimate local liquid heights in horizontal diabatic slug flow

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As the intermittent (slug) flow pattern was recently shown to be, along with the stratified flow regime, responsible for circumferential anisothermality of horizontal steam generating tubes operating at moderate steam qualities, the ability to estimate the local liquid levels in such tubes and to compare them with the position of the circumferentially maximum value of the externally applied heat loading appears to be of great practical importance for boiler designers. While the procedure of the estimation of minimum liquid heights (h_i) in horizontal stratified flows was suggested in previous papers, this study presents an analytical approach for engineering evaluations of h, in the horizontal, diabatic slug flow pattern. It is, importantly, shown that the use of the stratified flow-based approach to evaluate h, in slug flows results in the overestimation of actual liquid heights which may be detrimental for boiler tubes, especially under circumferentially nonuniform heat loading.

Keywords: two-phase flow; stratified flow; slug flow; circumferential anisothermality; circumferentially nonuniform heat fluxes; steam-generating tubes

Introduction

Currently, circumferential anisothermality of horizontal steam generating tubes represents a serious constraint in designing and operating certain types of industrial boilers (e.g., oncethrough steam generators, fluidized-bed-type generators, etc.). It would, therefore, be of great practical importance if the detrimental wall temperature profiles and yielded high thermal stresses which result in frequent failures of horizontal tubes² be minimized by their proper design. Recently, a number of studies 1-5 were presented which focused on physically-based design criteria intended to avoid local dryout in horizontal and slightly inclined pipes operating at low and moderate qualities. These studies were motivated by frequent encounters and discussions the authors had with representatives of industrial boiler manufacturing companies, both in the U.S. and abroad^{5,21} that raised the importance of a more physically-based design procedure of pipes operated under diabatic two-phase flow conditions. The present work represents, to a great extent, a continuation of these previous papers, especially those by Ruder, Bar-Cohen and Griffith.^{5,8} It is, therefore, desirable that, prior to reading this brief communication, one gets acquainted with the results discussed in the aforementioned studies.

It was important that, under certain operating and geometric conditions, it is not only the stratified flow pattern but also segments of the dispersed bubble and intermittent flow patterns that may result in tube overheating.¹⁻⁵ The authors experimentally found that the tube local dryout in the intermittent flow regime, which corresponds to the moderate quality range of 0.5-1% and is encountered in practical boiler applications more often than stratified flow² results from the insufficiently high slug formation/passage frequency. This yields the evaporation of a thin liquid film on the tube by an externally applied heat flux. Dimensionless bounding criteria, in the form of modified, liquid and gas superficial velocity-based Froude

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numbers, defining the flow and thermal conditions of circumferential anisothermality were developed and experimentally verified by Ruder, Bar-Cohen and Griffith, $1-4$ for circumferentially uniform heat fluxes, and tested by the same authors, for circumferentially nonuniform heat fluxes. A typical moderate quality anisotbermal region in the diabatic flow regime map is shown in Figure 1 to be bounded by the onset of the annular flow, from the right (Weisman *et al.*⁶; Bar-Cohen *et al.*^{1,2}), by the Ruder *et al.*^{$I-S$} criterion on the top, and by the diabatic boundary between the dispersed bubble and the intermittent flow regimes from the left. The latter was analytically defined by Dukler and Taitel.⁷

As most practical applications of modern boilers include circumferentially (and often axially) nonuniform patterns of heat loading, the ability to estimate local flow and thermal parameters appear to be of great engineering importance for most cases. For such conditions, the evaluation of possible dryout conditions is, to a great extent, reduced to the estimation of the local liquid level in the pipe and comparison of this level with the vertical position of the peak heat flux, $2¹$ as schematically shown in Figure 2.

The procedure of estimating local liquid heights in horizontal boiler tubes operating under diabatie stratified flow conditions was suggested in detail by Ruder¹ and Ruder, Bar-Cohen and Griffith. 3 This paper suggests an extension of this procedure to the case of horizontal, diabatic slug flows and, thus, seeks to provide boiler engineers with a more precise thermal design criterion for this type of steam generating tube.

Brief theoretical background and literature review--mechanisms governing stable slugs

Some of the recently published studies on adiabatic twophase flows have shed additional light on the physics of the horizontal slug flow phenomena. Importantly, it appeared that the difference between mechanisms governing the initiation of slugs and those responsible for the sustenance of developed slugs consecutively traveling through a horizontal pipe, results in the simultaneous existence of two distinct and unequal "critical" minimum values of the liquid level in the pipe (h_L) .

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The value of h_L which is associated with the *initiation* of slugs has been extensively studied, both analytically and experimentally. Taitel and Dukler⁹ applied an inviscid-flow approach to suggest that the liquid height in a pipe should reach a level

Figure 1 **Typical anisothermal region at moderate qualities in** diabatic flow regime map $(D=0.0254$ m, $p=1$ atm, $q''=10^6$ W/m²)

Figure **2 Typical liquid heights and possible heat flux profiles in case of a horizontal boiler tube operating in the slug flow regime**

of neutral stability so that a stable slug could develop from a solitary wave growing on the gas/liquid interface. The value of h_L is then determined from the expression describing the condition when the suction pressure generated over such a wave by the Bernoulli effect is large enough to overcome the stabilizing influence of gravity

$$
u_G = \left(1 - \frac{h_L}{D}\right) \left[\frac{g(\rho_L - \rho_G)A_G}{\rho_G S_i}\right]^{1/2} \tag{1}
$$

where the term $(1-h_L/D)$ is a factor based on experimental considerations. An ideal-inviscid flow approximation was also used in the analysis by Wallis and Dobson,¹⁰ Kordyban and Ranov, 11 Kordyban, 12 and others. Lin and Hanratty 13 used viscous type of the Kelvin-Helmholtz stability analysis (they analyzed the stability of a stratified flow to long wavelength waves) and also showed that in adiabatic, air/water systems, the neutral stability liquid level is required for the initiation of slugs. It is of interest to note that, for the conditions tested by Lin and Hanratty, the linear stability theory ceased to be applicable for high gas velocities, mainly due to nonlinear effects of coalescence of roll waves which could be accounted for neither by the viscous nor by the inviscid approaches mentioned above.

 $Lin¹⁴$ was the first to report that the liquid heights between consecutive, developed, undeeaying slugs traveling along horizontal pipelines most often appear to be much lower than the values of h_L theoretically predicted as a condition for the initiation of these same slugs. Lin's observations were further experimentally confirmed by Ruder, Hanratty and Hanratty.¹⁵ In accordance with the widely accepted slug model by Dukler and Hubbard,¹⁶ a moving slug sustains itself by accumulating liquid from the "carpet" just before its front. For a stable slug, the rate of "scooping up" liquid at the front equals the rate of shedding liquid from the tail. Alternately, if the amount of liquid lost by shedding through the tail exceeds that accumulated through the front, the slug would decay. Lin¹⁴ and Ruder, Hanratty and Hanratty¹⁵ concluded that the most general condition for the very existence of undeeaying slugs in a pipe is closely related to the amount of liquid available at the slug

front. In other words, for certain pipe geometry and fixed gas and liquid flow rates, there exists a minimum, or "critical", liquid film height in front of a moving slug for a horizontal slug flow to be possible.

Measurements of pressure fluctuations associated with moving liquid slugs (Lin and Hanratty¹⁷) as well as systematic observations of both the behavior of the inter-slug liquid film and typical visual manifestations of slugs enabled Ruder, Hanratty and Hanratty¹⁵ to model the slug front as a hydraulic jump. The behavior of the tail was, alternately, suggested to be governed by either a steady-state (Ruder, Hanratty and Hanratty¹⁵) or unsteady state (Jepson and Taylor²⁴) mechanism. As a result, it appeared to be possible to define a criterion for slug stability, a modified Froude number

$$
Fr = \frac{u_G - u_L}{\sqrt{gD}}\tag{2}
$$

whose "critical" values would determine the minimum prefront liquid heights. In Equation 2, u_G is the actual gas velocity and u_L is the actual velocity of the prefront liquid film. In the development of the slug model, the velocity distributions for both phases were assumed uniform. It should be noted that both models for the minimum values of h_L (that for the initiation of slugs and that for the existence of moving slugs) resulted from the *adiabatic flow* approach and were, consequently, experimentally verified for *adiabatic* systems. Corresponding charts where the adiabatic Froude numbers, Fr, were plotted against reduced prefront liquid heights, h_L/D , were given in Ruder, Hanratty and Hanratty (see figures 3 and 12).¹⁵ For adiabatic systems, the zone associated with stable or growing slugs was shown, in accordance with this approach, to be located to the right of the tail criterion curves. The minimum possible (for the adiabatic, air-water slug flow) values of prefront h_l/D were theoretically suggested and experimentally found to correspond to the range of Froude numbers determined by the limiting values of the two (steady and unsteady) tail criteria.

The present work represents an effort to expand these approaches to adiabatic case when a horizontal slug flow in a steam generating tube is a phenomena of flow boiling due to externally imposed heat loading (rather than merely a fullydeveloped concurrent flow of air and water) and, as a result, to give practical engineering guidelines to boiler designers seeking to predict minimum values of h_L . It must be kept in mind that the situation when there exist many slugs moving consecutively through a pipe at relatively high slug frequencies, is the one most often encountered in practical applications of both adiabatic pipelines (Scott, Shoham, Brill¹⁸) and steam generating tubes (Ruder, Bar-Cohen, Griffith⁵). Under such conditions, the height of the immediate prefront liquid layer would virtually represent the minimum height in the entire region in between two consecutive slugs. However, as distinct from adiabatic two-phase flows, diabatic flows are characterized by the change of flow parameters in the axial direction (along an operating locus as shown in Figure 1), which, in the case of boiling, results from the generation of bubbles in the liquid layer and from the consequent appearance of gas/liquid mixture. These effects were analyzed by Dukler and Taitel. 7 For example, in their study of the case of flow boiling, the liquid heights required by the inviscid approach for the slug formation were shown to be determined from

$$
u_G \ge \left(1 - \frac{h_L}{D}\right) \left[\frac{A_G(\rho_M - \rho_G)g}{\rho_G dA_M/dh_L}\right]^{1/2} \tag{3}
$$

where A_M is the area occupied by the gas-liquid mixture. However, the Dukler and Hubbard¹⁶ theory of moving slugs and Ruder, Hanratty and Hanratty¹⁵ model of the minimum

prefront h_L values necessary for the existence of undecaying slugs, both previously suggested for adiabatic flows, seem to remain valid for diabatic flows as well. Visual observations of slugs under adiabatic and diabatic conditions did not reveal remarkable differences in their shapes or other manifestations.¹ Moreover, while the occlusion of air into the bodies of moving slugs is inherent to air-water systems,¹⁵ the adiabatic conditions for slug stability that neglected the effect of aeration were experimentally found to do a good job in predicting the minimum liquid levels in the prefront regions of moderately aerated adiabatic slugs. This makes it possible to conclude that these conditions determining the values of interslug levels of liquid, may be extended, with certain modifications, for diabatic systems of horizontal boiler tubes.

Analysis--A method for the estimation of the minimum liquid heights

Following the previous section, as adiabatic slugs travel along the pipe, the interslug minimum liquid heights could be determined by the Froude number defined by means of Equation 2. Hence, it would be desirable if boiler designers be provided with a nomogram which would allow them to estimate local values of h_L on the basis of calculated values of corresponding *Fr.* However, in these cases, as distinct from adiabatic charts shown by Ruder, Hanratty and Hanratty (see figures 3 and 12 ¹⁵ where the phase velocities were considered constant, the Froude number must account for the local thermal and hydrodynamic parameters which vary in the streamwise direction of the locus in Figure 1, as a result of the change of vapor quality due to evaporation. Therefore, as distinct from adiabatic slug flows, in the case considered the liquid height values will not be constant but rather will vary along the same locus. Local h_L will depend on local flow and heat conditions and reach their minimum (limiting) value at the intersection point between the locus and the boundary of the annular flow. Thus, the goal is to redefine, for diabatic cases, the Froude number of Equation 2 to make the model applicable for conditions considered by boiler designers.

Therefore, it appears that the practical applicability of the method for the engineering evaluations of local critical values of h_L using the "diabatic" Froude number would depend on the ability to estimate local values of actual phase velocities in Equation 2. The actual gas velocity in the gas layer, u_G , which is considered to be close to the instantaneous front and tail velocity of a stable slug (Lin and Hanratty¹⁶) can be related to the total superficial gas velocity U_G , the latter being convenient for engineering calculations. As is mentioned above, in the case of flow boiling, the gas phase is not only concentrated in the gas layer, i.e., in the gas cavity between consecutive liquid slugs, but is also dispersed in the liquid film in the form of bubbles.

Taitel and Dukler⁷ showed that the total superficial vapor velocity is related to the superficial gas layer velocity, U_G^* , by means of a continuity equation

$$
U_G^* = U_G - U_L(1 - \phi)/\phi \tag{4}
$$

where ϕ is a void fraction. Further following Dukler and Taitel⁷ and using simple algebraic alterations and substitutions, one obtains an expression for the superficial gas velocity in the gas layer¹⁹

$$
U_G^* = U_G - U_L \frac{\bar{q}'' \pi D}{1.53 S_i h_{LG} \rho_G \left\{ \frac{\sigma_L (\rho_L - \rho_G) g}{\rho_L^2} \right\}^{1/4} - \bar{q}'' \pi D}
$$
(5)

where \bar{q}'' is the circumferentially average heat flux, and U_L and U_G are total superficial velocities of liquid and gas, respectively. Equation 5 is of primary importance because it represents a major step in the estimation of the gas phase velocity of Equation 2, while all parameters in its RHS are easily calculated or known in advance. The total superficial liquid velocity, U_G , and the superficial liquid velocity, U_L , can be calculated from thermal balances for saturated flow boiling in circular pipes²⁰

$$
U_G = \frac{4\bar{q}''L}{h_{LG}\rho_G D} \tag{6}
$$

$$
U_L = \frac{G D \mathbf{h}_{LG} - 4 \bar{q}'' L}{\rho_L D \mathbf{h}_{LG}}\tag{7}
$$

where L is the length of the pipe from the onset of net vapor generation at saturated boiling conditions (not just the length of the pipe from the onset of the slug flow regime) and G is the total mass flow rate of the mixture known in advance. The actual velocity of the gas layer can then be estimated by means of the mass balance as

$$
u_G \simeq U_G^* \frac{\pi D^2}{4A_G} \tag{8}
$$

Equation 8 is strictly valid for stratified flows, whereas for intermittent flows, where the voidage is a variable, it offers an approximation. It is, however, important to emphasize that this is a conservative approximation, since the appearance of liquid blocks bridging the entire pipe cross-section will lead to a decrease in the instantaneous void fraction in the pipe. This will cause an increase of actual u_G values as compared to those calculated by means of Equation 8. In accordance with the model by Ruder, Hanratty and Hanratty,¹⁵ a consequent increase of the critical Froude number (Equation 2) will lead to estimated $(h_L)_{min}$ values somewhat less than actual.

As for the actual liquid velocities, it was shown by Ruder, Hanratty and Hanratty¹⁵ that for U_G values close to those associated with the transition to the annular flow the actual velocities of the gas phase are much higher than those of the prefront liquid carpet. The latter can, therefore, be neglected in engineering estimations.

Equations 5 through 8 make it possible to redefine the modified Froude number (Equation 2) as

$$
Fr = U_{G}^{*} \frac{\pi D^{2}}{4A_{G}\sqrt{gD}}
$$
\n(9)

Since the calculation of *Fr,* using Equation 9 for the flow and thermal conditions of interest, would require the knowledge of certain geometrical parameters (A_G, S_i) , one would have to apply an iterative procedure.

A nomogram in the form Fr versus h_L/D , where Fr is defined in accordance with Equation 9, is given in Figure 3. As is shown in this figure, once the actual gas velocity has been estimated using Equations 5 and 8, the minimum possible values of h_L between the slugs can be easily obtained graphically. It is of importance to emphasize that the application of Figure 3 for the estimation of h_L results in the values of liquid heights different (lower) than the use of the stratified flow-based method previously suggested by Ruder, Bar-Cohen and Griffith.⁵ In other words, the latter yields an overestimation of local liquid heights. For example, for the atmospheric water/steam operating conditions of Bar-Cohen, Ruder and Griffith² with $\dot{G} = 440 \,\text{kg/m}^2 \cdot \text{sec}, q = 45 \,\text{kW/m}^2, \text{and } D = 0.0254 \,\text{m}, \text{the } (h_L/D)_{lim}$ appears to be 0.1-0.12 versus 0.546 (slug versus stratified); for the same flow rate but with $q'' = 10 \kappa W/m^2$, the $(h_L/D)_{lim}$ is $0.09-0.1$ versus 0.26. For the Rounthwaite²² conditions $(p=15 \text{ bar}; G = 290 \text{ kg/m}^2 \cdot \text{sec}; q'' = 65 \text{ kW/m}^2; D = 0.041 \text{ m}$),

Figure 3 Practical nomogram for boiler designers. Graphical **estimation of minimum interslug liquid heights in horizontal steam generating tubes**

 $(h_L/D)_{lim}$ equals 0.1-0.15 versus 0.314 and for the conditions of Styrikovich and Miropolski²³ ($p = 36$ bar, $G = 760$ kg/m² sec; $q'' = 100 \kappa W/m^2$; $D = 0.058$ m), $(h_L/D)_{lim}$ turns out to be 0.16-0.22 versus 0.36, present calculations versus stratified flow-based approach, respectively. Such an overestimation of the minimum interslug liquid level may be detrimental to boiler pipes.

It must be noted, once again, that while the present method of calculations yields minimum possible liquid heights in the slug flow, the local $(h_L/D)_{min}$ parameter is actually a complicated function of slug frequency. Unfortunately, analytical approaches to slug frequency calculations in diabatic flows have not, thus far, been developed.⁵ Therefore, for the needs of engineering calculations, it could be assumed that local liquid height values would actually fall within a range between two limits. While lower limit would be $(h_L/D)_{min}$, calculated in accordance with the present approach, the upper limit would be determined by the neutral stability theory. It is suggested that, for engineering evaluations, the latter limit be calculated according to Ruder, Bar-Cohen and Griffith. 5

Conclusion

An analytical approach for engineering estimations of minimum liquid heights in between consecutive slugs in horizontal boiler tubes has been developed. It has been suggested that for the local flow and thermal conditions of interest, the local liquid heights be estimated to fall between the neutral stability level and the minimum level in the prefront region of a moving slug. A practical nomogram for the graphical estimation of the minimum interslug liquid heights has been recommended for boiler designers.

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